MATHEMATICAL MODEL OF THE DRYING OF A POLYDISPERSE PRODUCT IN A PNEUMATIC TUBE-DRIER

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The differential equations describing the drying of a moist polydisperse product in a pneumatic drier are presented. A method for solving them numerically is described.

In [1] we described a mathematical model of the process of drying of a monodisperse product in a pneumatic tube drier, which adequately describes the actual drying process. This paper extends those investigations to describe the drying of a polydisperse product.

Unlike the model of the drying of a monodisperse product, in the model proposed here the kinetic equations of the heat and mass transfer and the equation of motion of the particles of the dried material are formulated for each fraction separately, and the balance equations for the moisture and heat are written for the whole mass of the dried material.

In this paper we consider the drying of a moist polydisperse product in a pneumatic gas drier. The well-known fractional composition of the dried material is assumed. The drying process is stationary. In any transverse cross section normal to the motion of the mixture of material and drying gas, all the parameters of the process are constant (the one-dimensional model). The temperature and moisture content over the cross section of the dried particles of material are constant. The diffusion and heat transfer in the flow along the motion of the mixture of gas and material is negligibly small compared with the amount of heat and mass of material transferred by the flow of mixture. The pressure drop is constant over the whole length of the apparatus.

Hence, we will consider the one-dimensional problem in the direction of the axis coinciding with the direction of motion of the mixture of material and drying gas. The heat and mass transfer obeys the Fourier–Fick laws.

At a distance x from the entrance to the tube drier we distinguish an element of volume of height dx and base of unit area. A change in the amount of moisture in the material of the i-th fraction in this volume $d[G_M\omega(\delta_i)/(1 - W(\delta_i))]W(\delta_i)$ occurs due to evaporation of moisture from the material in the drying gas $F(v_M(\delta_i))\beta(W(\delta_i))[P(T(\delta_i)) - p]dx$. Consequently, the equation of mass transfer between the i-th fraction of the material and the drying gas can be written in the form

$$\frac{d}{dx}\left[\frac{G_M\omega(\delta_i)}{1-W(\delta_i)}W(\delta_i)\right]+F(v_M(\delta_i))\,\beta(W(\delta_i))[P(T(\delta_i))-p]=0. \tag{1}$$

In the volume dx of the drying gas considered an amount of heat $F(v_M(\delta_i))\alpha(v_g, v_M(\delta_i), \theta)(\theta - T(\delta_i))dx$ is given to the material of the i-th fraction. Part of this heat is used to change the heat content of the material $d[G_M\omega(\delta_i)/(1 - W(\delta_i))]C_M(W(\delta_i))T(\delta_i)$ and part is used to evaporate the moisture (from the material into the gas) $F(v_M(\delta_i))\beta(W(\delta_i))[P(T(\delta_i)) - p]\cdot r(T(\delta_i))dx$. The equation of heat transfer between the drying gas and the dried material of the i-th fraction takes the form

$$\frac{d}{dx}\left[\frac{G_M\omega(\delta_i)}{1-W(\delta_i)}C_M(W(\delta_i))\,T(\delta_i)\right]+F(v_M(\delta_i))\,\{\alpha(T(\delta_i)-\theta)+\beta\,[P(T(\delta_i))-p]\,r(T(\delta_i))\}=0. \tag{2}$$

Here and everywhere henceforth the arguments in the functions α and β are omitted for simplicity.

In deriving the equation of motion of the particles of material of the i-th fraction we must take the variability of its mass into account. The derivation is similar to the

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derivation of the equation of motion of a particle in [1]. The equation has the following form:

$$v_{\text{M}}(\delta_i)\,\frac{dv_{\text{M}}(\delta_i)}{dx}+g-\frac{1}{m(\delta_i)}\,\Phi\,(\overrightarrow{v_g-v_{\text{M}}(\delta_i)})=0. \tag{3}$$

An amount $G_g d$ of moisture with drying gas and $\sum_{i=1}^{N}\frac{G_{\text{M}}\omega(\delta_i)}{1-W(\delta_i)}W(\delta_i)$ of moisture with material passes through each cross section of the tube drier. An amount $G_g d_b + [G_M/(1-W_b)]W_b$ of moisture with gas and material enters the drier. We obtain the following moisture-balance equation:

$$G_g d+\sum_{i=1}^{N}\frac{G_{\text{M}}\omega(\delta_i)}{1-W(\delta_i)}\,W(\delta_i)=G_g d_b+\frac{G_{\text{M}}}{1-W_b}\,W_b. \tag{4}$$

The subscript b denotes that the quantity concerned relates to the beginning of the tube drier.

An amount of heat $G_g C_g(d_b)\theta_b$ with gas and $[G_M/(1-W_b)]C_M(W_b)T_b$ of heat with material arrives in the drier. Part of this heat emerges from the tube drier with material and gas $\sum_{i=1}^{N}[G_M\omega(\delta_i)/(1-W(\delta_i))]C_M(W(\delta_i))T(\delta_i) + G_g C_g(d)\theta$, and part $\sum_{i=1}^{N} G_g(d-d_b)r(T(\delta_i))$ is used to evaporate moisture from the material in the gas. Taking the loss of heat through the walls Q_l into account, we obtain the following heat-balance equation:

$$\begin{aligned}&G_g C_g(d)\,\theta+\sum_{i=1}^{N}\frac{G_{\text{M}}\omega(\delta_i)}{1-W(\delta_i)}\,C_{\text{M}}(W(\delta_i))\,T(\delta_i)+\\&+\sum_{i=1}^{N}G_g(d-d_b)\,r(T(\delta_i))+Q_l=G_g C_g(d_b)\,\theta_b+\frac{G_{\text{M}}}{1-W_b}\,C_{\text{M}}(W_b)\,T_b\,.\end{aligned} \tag{5}$$

In this equation

$$Q_l=\frac{\Lambda}{S}\,K\int_0^x[\theta(\zeta)-T_s\,]\,d\zeta.$$

We will formulate the mathematical problem of describing the drying process in the pneumatic drier as follows: we wish to determine the values of $W(\delta_i)$, $T(\delta_i)$, $v_M(\delta_i)$, θ and d which satisfy Eqs. (1)-(5) (along the length of the apparatus and at the exit from it), if we know the initial conditions of the process

$$\begin{aligned}&W(\delta_i)\,|_{x=0}=W_b,\; T(\delta_i)|_{x=0}=T_b,\\&v_{\text{M}}(\delta_i)\,|_{x=0}=v_{\text{M},b},\; d\,|_{x=0}=d_b,\;\theta\,|_{x=0}=\theta_b.\end{aligned} \tag{6}$$

The function $P(T(\delta_i))$ in (1) and (2) is found from tables or analytically.

The surface of the material of the i-th fraction $F(v_M(\delta_i))$ in unit volume of the mixture, which occurs in Eqs. (1) and (2), is expressed as follows in terms of the average dimensions of the i-th fraction of material δ_i and the velocity $v_M(\delta_i)$:

$$F\,(v_{\text{M}}(\delta_i))=\frac{6}{\delta_i}\cdot\frac{G_{\text{M}}\omega\,(\delta_i)}{\rho_{dm}v_{\text{M}}(\delta_i)}\,,$$

while the numerical value of the aerodynamic resistance force of the medium $\Phi(\overrightarrow{v_g - v_M(\delta_i)})$ is given by [2]

$$\xi\,\frac{\pi\delta_i^2}{8}\,\rho_g(\theta,\,d)(v_g-v_{\text{M}}(\delta_i))^2.$$

We substitute these quantities into Eqs. (1)-(5) and multiply the first three equations by δ_i. Assuming that the mass of a particle of material of the i-th fraction $m(\delta_i) = (\pi\delta_i^3/6)\rho_M(W(\delta_i))$, we obtain a system in which the multiplier for the derivative is δ_i. Since, for finely dispersed products (precisely those which are being dried in the pneumatic driers) the value of δ_i is much less than the value of all the remaining parameters of the system, Eqs. (1)-(5) belong to the class of system of equations with a small parameter in the derivatives [3]. A characteristic feature of such systems is the presence in the neighborhood of the point $x = 0$ of a region whose dimensions are of the order of δ_i $(0 \leq x \leq O(\delta_i))$; in this region the solution of the system (here $W(\delta_i)$, $T(\delta_i)$, $v_M(\delta_i)$) changes considerably. This region is called the boundary layer, for which each fraction of material has its own value. The integration step in the region of the boundary $x = 0$ (the origin of the apparatus) should be considerably less than the thickness of the boundary layer [4]. Hence, when choosing the constant step over the whole integration region this fact leads to a considerable increase in the amount of calculation required for small values of the parameters for the derivatives (in this case δ_i). It is better to use an integration step which varies along the length of the tube drier, and which depends on the behavior of the solution of the initial problem. In the region of the boundary it should be much less than δ_i, while outside the boundary layer the step can be chosen to be constant, not exceeding a certain value H_0.

By replacing the derivatives by constant ratios we can reduce Eqs. (1)-(5) to a system of nonlinear algebraic equations. In particular, instead of (1) we will consider the equation

$$\frac{1}{h_j}\left[\frac{W_{j+1}(\delta_i)}{1-W_{j+1}(\delta_i)}-\frac{W_j(\delta_i)}{1-W_j(\delta_i)}\right]+\frac{6}{\delta_i}\,\frac{\beta_j}{\rho_{\mathrm{dm}}v_{\mathrm{M},j}(\delta_i)}\left[P\left(T_j(\delta_i)\right)-p_j\right]=0. \qquad (7)$$

Here $h_j = x_{j+1} - x_j$ and the subscript j or j + 1 denotes that the value of the corresponding function is taken at the point x_j or x_{j+1}.

Knowing the values of all the functions at the point x_j, from (7) for i = 1 (i is the number of the fraction) we find $W_{j+1}(\delta_1)$; from the remaining two finite-difference equations (not given here) we find $T_{j+1}(\delta_1)$ and $v_{M,j+1}(\delta_1)$. Changing the value of i from 1 to N we find the values of the moisture content, the temperature, and the velocity of the material for all the fractions (for all δ_i). Then, from (4) we find the moisture content of the gas d at the point $x_{j+1}(d_{j+1})$, and from (5) we find θ_{j+1}.

By choosing h_{j+1} and repeating this process we find $W(\delta_i)$, $T(\delta_i)$, $v_M(\delta_i)$, d and θ at the point x_{j+2} etc.

Thus we find the distribution of all the parameters of the drying process along the length of the tube drier. Hence we find not only the values of the moisture content, the temperature, and the velocity of the material of all the fractions, but also their "averaged" values. For example, the mean temperature of the material at the point x_j is determined as follows. We find the total amount of heat Q which passes with the material of all the fractions through the cross section x_j

$$Q=\sum_{i=1}^{N} G_{\mathrm{M}}\omega(\delta_i)\,C_{\mathrm{M}}\left(W_j(\delta_i)\right)T_j(\delta_i).$$

On the other hand, $Q = G_M C_M(W_{j,\mathrm{av}})T_{j,\mathrm{av}}$, whence

$$T_{j,\mathrm{av}}=\frac{Q}{G_{\mathrm{M}}C_{\mathrm{M}}\left(W_{j,\mathrm{av}}\right)}.$$

The average moisture content of the material $W_{j,\mathrm{av}}$ required to determine $T_{j,\mathrm{av}}$ can be found from the equation

$$W_{j,\mathrm{av}}=\frac{\sum_{i=1}^{N} G_{\mathrm{M}}\omega(\delta_i)\,W_j(\delta_i)}{G_{\mathrm{M}}}.$$

To study the qualitative behavior of the parameters of the drying process of a polydisperse product we carried out a series of computer calculations. As an example we will present the calculation of the drying of sodium fluosilicate having six fractions, with an average diameter of the fractions of 1.25, 0.9, 0.8, 0.71, 0.63, and 0.5 mm and a percentage content of dry material of each of these fractions of 2.44, 5.8, 11.2, 20.1, 24.5, and 35.96% respectively. Figure 1 shows the average moisture content of the material (curve 1) and the moisture content of four fractions along the length of the tube drier. It can be seen that particles of the material having maximum dimensions (curve 2) are practically not dried, while particles of material with minimum dimensions (5) are completely dried inside the apparatus. Curves 3 and 4 show the changes in the moisture content of the material of intermediate fractions (0.8 mm and 0.63 mm) along the length of the tube. Figure 2 shows the dependence of the velocity of the drying gas (curve 1), the average velocity of the materials

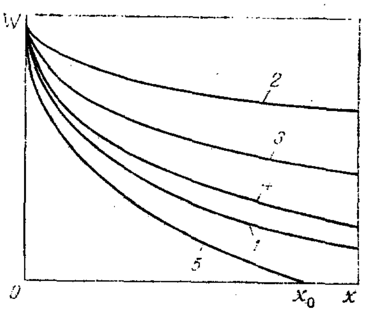

Fig. 1.

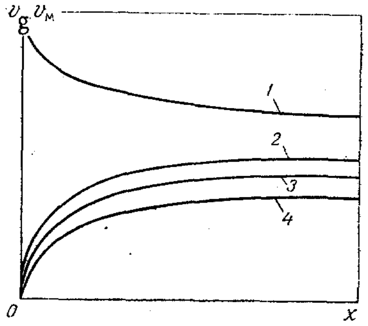

Fig. 2.

Fig. 1. Dependence of the moisture content of the material of different fractions on the length of the tube drier: 1) W_{av}, 2) W(0.8), 3) W(1.25), 4) W(0.63), 5) W(0.5).

Fig. 2. Profiles of the velocity of the gas and material of different fractions: 1) v_g, 2) $v_{M,av}$, 3) $v_M(0.5)$, 4) $v_M(1.25)$.

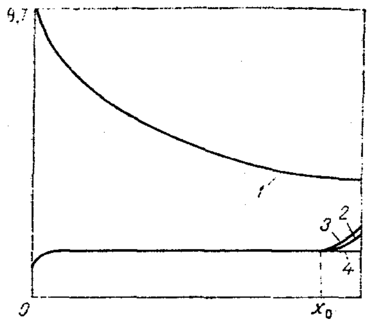

Fig. 3

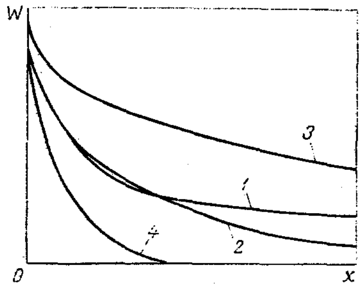

Fig. 4

Fig. 3. Graphs of the temperature of the gas and material of different fractions: 1) θ; 2) T_{av}; 3) T (0.5); 4) T (1.25).

Fig. 4. Dependence of the moisture content of the material on the length of the apparatus for different models of the drying process: 1,3,4) the polydisperse model (W_{av}, W (3.4), W (1.7); 2) the monodisperse model.

(2), the velocity of the maximum fraction of material (4) and the velocity of the minimum fraction of the material (3) and the length of the apparatus. The change in the temperature of the drying gas (1) and the temperature of the material of the different fractions along the length of the tube are shown in Fig. 3. Curve 2 describes the change in the average temperature of the material, while curves 3 and 4 describe the change in the temperature of its minimum and maximum fractions. The variation in the form of the curves 2 and 3 after the point $x = x_0$ is due to the fact that the material of the minimum fraction gives, in the part of the tube from 0 to $x = x_0$, all of its moisture to the drying gas and begins to heat up. As a result the average temperature of the material increases.

To check the model (1)-(6) we carried out computer calculations of the drying of Moscow coal [5]. The results obtained are shown in Table 1. We can conclude from a comparison of the experimental data and the data obtained from the model that the model which has been constructed of the drying of a polydisperse product satisfactorily describes the actual drying process occurring in the pneumatic drier.

Numerical calculations of the drying process enable us to conclude that the polydisperse model of drying more accurately describes the actual process than the monodisperse model with an equivalent particle diameter, determined by some averaging method.

TABLE 1. Comparison of the Experimental and Theoretical Data on the Drying of Moscow Coal (at the exit of the drying tube)

Expt. No.	W, %		T, °C		θ, °C	
	Expt.	Theory	Expt.	Theory	Expt.	Theory
1	27,9	28,2	69,7	76,8	230	260,9
2	26,8	27,5	67,4	62,3	224	239,2
3	23,4	22,4	68,6	79,4	238	250
4	27,2	28,5	68,8	76,4	204	240
5	23,4	22,06	69,9	73,3	163	149,7

For a more accurate answer to the question of when it is best to use the polydisperse model and when one can use the simpler monodisperse model we carried out a series of computer calculations. It was found that if the diameters of the particles of the dried material differ considerably, there are considerable differences in the results obtained using the monodisperse and polydisperse models. We calculated a model problem of the drying of sodium fluosilicate having two fractions (3.4 mm and 1.7 mm) with the same content by weight of material of each of them. Figure 4 shows the results obtained by calculation for the moisture content of the material using the polydisperse model (curve 1) and the monodisperse model (curve 2) (the equivalent diameter was found by a linear and volume-surface method of averaging [2]; the latter is more accurate for calculations of the drying process). Curves 3 and 4 show the change in the moisture content of the first and second fractions of the material along the length of the drying tube. For this example the final moisture contents of the product calculated using the polydisperse and monodisperse models were 4.2% and 1.2% for an initial moisture content of 20%.

If the differential curve of the distribution of the mass of the moist product has an excess ≥ 2, and a variation $\leqslant 30\%$ (i.e., this curve has a pronounced maximum and then drops sharply) we can use an equivalent particle diameter and carry out the calculations using the monodisperse model. In this case the main mass of the particles has a single size.

In conclusion it should be noted that for a number of fractions of the material equal to 1, this model changes into the model of the drying of a monodisperse product [1].

Hence, it is desirable to use the polydisperse model of drying when the diameters of the particles of the material differ considerably, while the content by weight of these fractions is appreciable. In addition, it is necessary to use this model when investigating the drying of products for which it is necessary to know the change in the parameters of the material (the moisture content and the temperature) along the length of the drier for all the fractions. In particular, by studying the drying process of labile materials one can estimate the fraction of the material decomposed during the drying, by knowing the temperature of the material of all the fractions. The polydisperse model must also be used to calculate the drying of products which deteriorate in quality by giving off water of crystallization. The fraction of this "poor" product can be estimated only using the polydisperse model.

NOTATION

G_M, G_g, weight flow rates of the absolutely dry material and the drying gas through a unit cross section of the tube in unit time; $W(\delta_i)$, d, moisture contents of the material of the i-th fraction and the gas; $T(\delta_i)$, θ, temperatures of the material of the different fractions and the gas; $\omega(\delta_i)$, relative content (in percentage by weight) of the material of the different fractions; α, β, heat and mass transfer coefficients; $P(T)$, equilibrium vapor pressure of water at the temperature T; p, partial pressure of the water vapor in the gas; $C_M(W)$, $C_g(d)$, heat capacities of the material for a moisture content W and of the gas for a moisture content d; $r(T)$, specific heat of vaporization at the temperature T; g, acceleration due to gravity; $v_M(\delta_i)$, v_g, linear velocities of the material and the gas; Λ, S, length of the perimeter and the area of cross section of the tube drier; K, heat transfer coefficient from the drying gas through the walls of the tube to the surrounding medium with temperature T_s; ρ_{dm}, $\rho_M(W)$, $\rho_g(\theta, d)$, densities of the dry material, the material with a moisture content W, and the drying gas at a temperature θ and a moisture content d, respectively; and ξ, the drag coefficient of a particle.

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MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER IN A MELT DURING INTERACTION WITH JETS OF A CHEMICALLY ACTIVE GAS

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The temperature and concentration distributions of reacting components of a melt moving in an axisymmetric bath are obtained during interaction with jets of a gas injected on its surface.

The velocity distribution of a melt in an axisymmetric bath is found in [1] for the delivery of a gas jet to the free surface. It is possible to solve the problem of heat and mass transfer in a melt reacting with a gas jet for a known velocity field in order to obtain data on the characteristics of the process and on the efficiency of the reactor (oxygen converter, pyrolysis apparatus, etc.).

The physical model of jet interaction with a melt is represented in Fig. 1. There is a surface L on the lune on which the gas jet reacts with the melt. The heat being liberated is expended in heating and evaporating its components. Because of the formation of a high-temperature zone, the interface is heated to a temperature exceeding the temperature of the melt. The heated layer of the melt together with the oxides are transferred from the lune over the volume of the bath. As soon as the necessary thermodynamic state is achieved, reactions between the oxides and the melt components take place, including the appearance of gaseous interaction products. Therefore, the dimensions of the reaction surface, its temperature, and the concentration of the chemically active reaction products in combination with the hydrodynamic circumstances predetermine the rate of the processes in the bath.

In constructing the mathematical model, we assume that the purging mode does not change, and the motion is self-similar, i.e., with a turbulent analog for the Reynolds number not less than the limit value for the reactor [1]. Such an assumption permits considering the steady-state velocity field in the bath. The component concentrations in the melt and its temperature because of the chemical reactions are time dependent, hence the heat and mass transfer process is a nonstationary process.

The equation for the change in component concentration in a melt must be solved in conjunction with the energy equation governing the temperature change. These equations are written thus in the presence of chemical reactions [2]:

$$\frac{dm_n}{d\tau} = D_T \nabla^2 m_n + \Delta m_n;$$

$$\rho \frac{dh}{d\tau} = \lambda_T \nabla^2 T + D_T \operatorname{div}\left(\sum_n h_n \rho \operatorname{grad} m_n\right) + \Delta h.$$

The change in concentration of the n-th component because of chemical reactions is taken into account by Δm_n while Δh is the quantity of heat being liberated or absorbed per unit time. An assumption about the constancy of the turbulent analogs of the coefficients of diffusion and heat conduction over the volume of the path is introduced for the conditions of developed turbulent motion, which permitted it to be extracted outside the differentiation symbol. For

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